The Philosophical Significance of Applying Set-Theoretic Methods in Other Mathematical Domains

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Abstract: In mathematical practice, the application of one mathematical domain within another is widely understood as a form of "internal application." Just as mathematical techniques are applied outside mathematics, such as in the sciences, internal applications within mathematics itself are also philosophically significant. They prompt questions about the purity of methods and the nature of explanatory proofs. In this talk, I will explore some of the philosophical dimensions of internal applications within mathematics, focusing specifically on the use of set-theoretic methods in other mathematical domains. This approach reveals essential connections to foundational questions in the philosophy of mathematics.

A central philosophical question in set-theoretic foundations is whether we can justify new axioms to settle statements that are independent of the standard theory, such as the continuum hypothesis. Here, views diverge. Absolutist practitioners argue that it is possible, through the addition of extrinsically justified axioms like projective determinacy. Pluralist practitioners, on the other hand, argue that this is not possible, given the numerous alternative axioms, none of which can be deemed definitively preferable. Interestingly, both perspectives invoke set theory's foundational role and its relevance to other mathematical domains. I will argue that resolving this philosophical tension hinges on the effectiveness of set-theoretic methods when applied in other mathematical domains.