



Deep disagreement in set theory

Deborah Kant

University of Konstanz

deborahkant.org

15. April, 2021

Early Lunch Philosophy, Konstanz



Volkswagen**Stiftung**

Universität
Konstanz





Part I: A case of deep peer disagreement among set theorists

Disagreement among set theorists

Arguments for *deep* disagreement

Arguments for *peer* disagreement

Part II: Remaining steadfast or suspending judgement?

Rationality and epistemic goals

Aiming at set-theoretic progress

Leaving the set-theoretic research context

Summing up



Part I: A case of deep peer disagreement among set theorists

Disagreement among set theorists

Arguments for *deep* disagreement

Arguments for *peer* disagreement

Part II: Remaining steadfast or suspending judgement?

Rationality and epistemic goals

Aiming at set-theoretic progress

Leaving the set-theoretic research context

Summing up



Pluralist and absolutist framework beliefs

Pluralist framework beliefs:

- No new axioms will and should be adopted.
- ZFC is sufficient to solve all set-theoretic problems.

Absolutist framework beliefs:

- New axioms will and should be adopted.
- ZFC is not sufficient to solve all set-theoretic problems.



Part I: A case of deep peer disagreement among set theorists

Disagreement among set theorists

Arguments for *deep* disagreement

Arguments for *peer* disagreement

Part II: Remaining steadfast or suspending judgement?

Rationality and epistemic goals

Aiming at set-theoretic progress

Leaving the set-theoretic research context

Summing up



Deep disagreement according to Fogelin's account

Normal argumentative context

Two individuals share many background beliefs and preferences, which are not themselves the subject of the argumentative exchange. (Fogelin 2005 (1985))

Deep disagreement

*We get a deep disagreement when the argument is generated by a clash of **framework propositions**. (p. 8)*

Systematicity

*[W]e do not simply find isolated propositions . . . , but instead a **whole system of mutually supporting propositions**. (p. 9)*



Deep disagreement according to Fogelin's account

Deep disagreement among set theorists

The disagreement between the pluralist and the absolutist is a systematic disagreement over framework propositions, which are not themselves the subject of set-theoretic research activity.

Pluralist framework beliefs:

- No new axioms will and should be adopted.
- ZFC is sufficient to solve all set-theoretic problems.

Absolutist framework beliefs:

- New axioms will and should be adopted.
- ZFC is not sufficient to solve all set-theoretic problems.



Deep disagreement according to Lynch's account

Fundamental epistemic principles

A fundamental epistemic principle is an implication which takes as a premise a descriptive fact and as conclusion a doxastic state.

Example

If there is a (mathematical) proof that p_m , then you are justified in believing that p_m .

Example

Testimony: If you are justified in believing that S is reliable and S asserts that p , then you are prima facie justified in believing that p

Deduction: If you are justified in believing that p and that p entails q , then you are justified in believing that q . (Matheson 2018, p. 3)



Deep disagreement according to Lynch's account

Pluralist epistemic principle

If p_m is proven independent from ZFC, then you are justified in believing that p_m is not about a matter of fact.

Absolutist epistemic principles

- If p_m solves many questions outside of set theory in the way that practitioners predict and leads to a coherent theory, then you are justified in raising your degree of belief that p_m .
- If p_m implies generic absoluteness of some important theory, then you are justified in raising your degree of belief that p_m .
- If many unrelated statements imply p_m , then you are justified in raising your degree of belief that p_m .



Deep disagreement according to Lynch's account

Four conditions for a deep disagreement by Lynch

- Commonality** The pluralist and the absolutist share the **common epistemic goal** of set-theoretic progress.
- Competition** The absolutist accepts extrinsic justification of axioms as a **reliable method**, while pluralists only accept ZFC-proofs as a reliable method. This **leads to incompatible beliefs**, e.g. regarding PD.
- Non-arbitration** Although both the pluralist and the absolutist accept ZFC-proofs as a reliable method, there are **no more fundamental principles** that settle the disagreement.
- Circularity** “The epistemic principle in question **can be justified only by means of an epistemically circular argument.**”
(Lynch 2010, p. 265)



Deep disagreement according to Lynch's account

No circularity

Set-theoretic epistemic principles are not mathematical in nature but apply to mathematical statements. Therefore, they **cannot be justified by a circular argument**.

Fundamentality with respect to the set-theoretic research context

Set-theoretic epistemic principles **cannot be justified by means inside the set-theoretic research context**. Therefore, I assume them to be fundamental with respect to the set-theoretic research context.



Part I: A case of deep peer disagreement among set theorists

Disagreement among set theorists

Arguments for *deep* disagreement

Arguments for *peer* disagreement

Part II: Remaining steadfast or suspending judgement?

Rationality and epistemic goals

Aiming at set-theoretic progress

Leaving the set-theoretic research context

Summing up



Peer disagreement

Epistemic peerhood

Two individuals are epistemic peers regarding p if they are in **the same epistemic position regarding p** , i.e., the probability that they are correct about p is the same.

Example

By contrast, if two individuals, Anna and Bea, disagree over a proposition p and Bea is in **a better epistemic position regarding p** , Anna should **adapt her degree of belief towards Bea's** and Bea should **remain steadfast**. Bea has probably considered more evidence, spent more time evaluating her evidence, or was more focused and clear-minded when evaluating her evidence.



Peer disagreement

Unknown epistemic position

In most real-world scenarios, the epistemic position of an individual is unknown.

No epistemic superiority = epistemic peerhood

As long as one does not know which of the two individuals is in an epistemic superior position, the case can be treated like a case of epistemic peerhood.

Set-theoretic epistemic peers

The relevant evidence regarding the framework propositions is set-theoretic and philosophical. It is plausible to assume that there are a pluralist and an absolutist who are similarly aware of the current state of research, and have similar philosophical expertise (e.g. Woodin and Hamkins).



Deep peer disagreement

Pluralist

epistemic
peers

Absolutist

FRAMEWORK BELIEFS

No new axioms will and should be adopted.

ZFC is sufficient to solve all set-theoretic problems.

Fundamental epistemic principles

Pluralist epistemic principle

Deep
disagreement

FRAMEWORK BELIEFS

New axioms will and should be adopted.

ZFC is not sufficient to solve all set-theoretic problems.

Fundamental epistemic principles

Absolutist epistemic principles



Part I: A case of deep peer disagreement among set theorists

Disagreement among set theorists

Arguments for *deep* disagreement

Arguments for *peer* disagreement

Part II: Remaining steadfast or suspending judgement?

Rationality and epistemic goals

Aiming at set-theoretic progress

Leaving the set-theoretic research context

Summing up

Rational response to deep peer disagreement

Question

What is the rational response for the pluralist and the absolutist facing this deep peer disagreement?

Lougheed: remain steadfast

*If [an individual] really could somehow reasonably believe that **epistemic benefits** are in the offing within a research context, then she would be rational to remain steadfast. (Lougheed 2018, p. 266)*

Matheson: equal weight view (EWV)

*If EWV is true, then both parties in a peer disagreement should split the difference ... [In the case of peer as well as deep disagreement] there are **true epistemic principles** that govern the situation, and ... individuals should believe according to those principles. (Matheson 2018, p. 9)*



Part I: A case of deep peer disagreement among set theorists

Disagreement among set theorists

Arguments for *deep* disagreement

Arguments for *peer* disagreement

Part II: Remaining steadfast or suspending judgement?

Rationality and epistemic goals

Aiming at set-theoretic progress

Leaving the set-theoretic research context

Summing up



Rationality in terms of epistemic goals

Epistemic goals

Set-theoretic progress is a **group epistemic goal** of the set-theoretic community and true beliefs is an **individual epistemic goal**.

Rationality in terms of set-theoretic progress

A response is rational iff it is **beneficial to set-theoretic progress**.



Part I: A case of deep peer disagreement among set theorists

Disagreement among set theorists

Arguments for *deep* disagreement

Arguments for *peer* disagreement

Part II: Remaining steadfast or suspending judgement?

Rationality and epistemic goals

Aiming at set-theoretic progress

Leaving the set-theoretic research context

Summing up



In favour of remaining steadfast

Part of the Epistemic Benefits of Deep Disagreement Argument (Lougheed)

*S reasonably believes that **there are future epistemic benefits** from continuing to believe P within the [research] context of R in the face of peer disagreement about P . (Lougheed 2018, p. 283)*

Epistemic benefits

Lougheed argues that diverse groups are **better at problem-solving and prediction-making** by support of an empirical study by Page (2007) (pp. 269ff), that disagreement works **against confirmation bias** (pp. 271f) and encourages each party to **find strong arguments** (pp. 272ff).



In favour of remaining steadfast

Diverse mathematics are epistemically beneficial

Allowing for deep disagreement is one way to guarantee diverse mathematics, because **mathematical activities are influenced by framework beliefs** (Rittberg 2020), and diverse mathematics are epistemically beneficial (mathematics should develop in all different valuable directions).



In favour of remaining steadfast

Diverse mathematics are not epistemically detrimental

- **Collaborations** between deeply disagreeing set theorists are possible. Examples are Woodin and Hamkins (4 co-authored papers), and Foreman, Magidor, Shelah (1988).
- Deeply disagreeing set theorists **value each other's mathematical work** (based on interview study).

"when I look at Woodin's work, then I see very difficult and interesting work ... But when he says this is [a] justification for the argument that this type of thing is certainly true, instead I see a justification of this type of theory is very convenient from [a] certain point of view."

"I think Hamkins is very brilliant ... I'm not convinced by his philosophical views on set theory and the multiverse. But I have the greatest respect for his mathematical work."



Against suspending judgement

The equal weight view

Both beliefs should be given equal weight; a peer disagreement is a symmetric situation, and the parties should split the difference (of their degrees of belief). This results in a situation of agreement.

Example

If Anna believes P and Bea believes non-P, the equal weight view requires that both **suspend judgement** on P.

Example

Restaurant check case (Christensen 2007): Two friends (epistemic peers) calculate different values to divide the check after dinner, and find out that they disagree. In that case, they should give equal weight to both beliefs.



Against suspending judgement

Suspending judgement

The equal weight view requires that the pluralist and the absolutist should suspend judgement on their framework beliefs.

Suspending judgement would be epistemically detrimental

- Guidance of research activities by framework beliefs would be lost, and probably some mathematical developments would be blocked.
- The research would instead be guided by values on which peers agree, which would increase homogeneity of set-theoretic research.
- On a personal level, a re-orientation towards an alternative guidance of set-theoretic research activities is probably strenuous, e.g. questions which appeared attractive become less attractive.



Part I: A case of deep peer disagreement among set theorists

Disagreement among set theorists

Arguments for *deep* disagreement

Arguments for *peer* disagreement

Part II: Remaining steadfast or suspending judgement?

Rationality and epistemic goals

Aiming at set-theoretic progress

Leaving the set-theoretic research context

Summing up



Aiming at true beliefs

Matheson's argument

If the equal weight view is true, individuals should believe according to this principle.

True beliefs about framework propositions rather than set-theoretic progress

If the pluralist and the absolutist aim at true beliefs about the framework propositions and no longer at set-theoretic progress, the arguments given in favour of remaining steadfast collapse. Hence, if the equal weight view is true, then they should suspend judgement on the framework propositions.



Set-theoretic progress and true beliefs

Can they afford to be wrong?

- If the pluralist and the absolutist **aim at true beliefs** about framework propositions (individual epistemic goal), **they cannot afford to be wrong**.
- If they **aim at set-theoretic progress** (group epistemic goal), then **they can afford to be wrong** about the framework propositions, because remaining steadfast is beneficial to set-theoretic progress.

Set-theoretic progress

As part of aiming at set-theoretic progress, the set-theoretic community aims at true beliefs about set-theoretic propositions but not about framework propositions. In other words, the set-theoretic community wants to find out, for example, whether (*) is compatible with MM^{++} rather than whether independent sentences are about matters of fact.



Set-theoretic progress and true beliefs

Matheson's argument

Even if the equal weight view has epistemically detrimental consequences, if the equal weight view is true, individuals should believe according to this principle.

My argument

Even if the equal weight view is true, if the group epistemic goal of set-theoretic progress is superior to the individual goal of true beliefs about framework propositions, the pluralist and the absolutist should remain steadfast on the framework beliefs, because the equal weight view has epistemically detrimental consequences if applied to the framework propositions.



Part I: A case of deep peer disagreement among set theorists

Disagreement among set theorists

Arguments for *deep* disagreement

Arguments for *peer* disagreement

Part II: Remaining steadfast or suspending judgement?

Rationality and epistemic goals

Aiming at set-theoretic progress

Leaving the set-theoretic research context

Summing up



Overview on the argument

