Set-theoretic independence approached by an interview study

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Set-theoretic Independence

- Three Notions of Truth
- Independent Sentences
- Philosophical Questions

Interview Study

Three Disciplines: Philosophy, Mathematics and Social Science

Truth in a Model

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Given a set M, we can define (recursive definition in set theory) for a sentence φ of $\mathcal{L}_\in$ the binary relation 'φ is true in M':

$$M \models \varphi.$$

Example

Let $\varphi = \exists x \forall y (\neg y \in x)$ and $M = \omega$. Then $M \models \varphi$.

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Mathematical Account: Provability in ZFC

Definition (Logical Consequence of ZFC)

φ is a *logical consequence* of ZFC, ZFC $\models \varphi$, iff for all M (M $\models$ ZFC $\Rightarrow$ M $\models \varphi$).

Theorem (Gödel's Completeness Theorem for ZFC)

ZFC $\models \varphi \Leftrightarrow$ *ZFC* $\vdash \varphi$.

Mathematical Convention

Many mathematicians accept ZFC as a foundation of mathematics.

Mathematical Account of Truth in Mathematics

If ZFC $\vdash \varphi$ (equivalently if ZFC $\models \varphi$), then φ is true in mathematics.

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Truth in a Standard Model

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If there is a standard model $\mathcal{M}$ of a subject matter, then we can define for a sentence φ of the respective language:

$$\varphi \text{ is true iff } \mathcal{M} \models \varphi.$$

Example

The subject matter is the natural numbers, $\mathcal{L} = \mathcal{L}_{\mathrm{PA}} = \{\mathbf{0}, \mathbf{S}, +, \cdot, <\}$, $\mathcal{N} = (\mathbb{N}, 0, \mathrm{S}, +, \cdot, <)$, and $\varphi = \forall x \, \exists y \, (x < y)$. Then, φ is true, because φ is true in $\mathcal{N}$.

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Subject Matter:	Number Theory	Set Theory
Axioms:	Peano Axioms	ZFC Axioms
Standard Model:	Yes: $\mathcal{N}$	The set-theoretic universe $\mathcal{V}$? (undefinable)
Meta Theory:	Set Theory	Set Theory
Independence:	Only for meta-theoretic sentences	For many set-theoretic sentences

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Definition: Independent Sentence

Definition (Independent Sentence)

Let φ be a sentence of $\mathcal{L}_{\in}$. We say that φ is *independent* iff we can prove $\text{Con(ZFC)} \Rightarrow \text{Con(ZFC} + \varphi)$ and $\text{Con(ZFC)} \Rightarrow \text{Con(ZFC} + \neg\varphi)$.

Assuming that there is a model of ZFC, there is a model of ZFC $+ \varphi$ *and another model of ZFC* $+ \neg\varphi$.

Remark

This is equivalent to: φ is independent iff $\text{Con(ZFC)} \Rightarrow \text{ZFC} \nvdash \neg\varphi$ and $\text{Con(ZFC)} \Rightarrow \text{ZFC} \nvdash \varphi$.

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Inner Models

Definition (Inner Model)

An *inner model* of ZFC is a transitive class that contains all ordinals and in which ZFC holds.

Remark

An inner model is a proper class.

Inner Model Template

Let C be an inner model. If $\mathsf{ZFC} \vdash (\mathsf{ZFC} + \varphi)^{\mathsf{C}}$, then $\mathrm{Con}(\mathsf{ZFC}) \Rightarrow \mathrm{Con}(\mathsf{ZFC} + \varphi)$.

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Forcing

Forcing Template

Start with a (countable, transitive) model M of ZFC: Assume Con(ZFC).
Find a partial order (in M) and a generic object G (outside of M), such that $M[G] \models \neg\varphi$. Since M is still a model of ZFC, we can conclude $\text{Con}(\text{ZFC} + \neg\varphi)$.
Thus, we have proven $\text{Con}(\text{ZFC}) \Rightarrow \text{Con}(\text{ZFC} + \neg\varphi)$.

Example 1: GCH

General Continuum Hypothesis (GCH)

For every infinite cardinal κ: $\kappa^+ = 2^\kappa$.

Independence of GCH

Gödel's L: $\text{Con(ZFC)} \Rightarrow \text{Con(ZFC} + \text{GCH)}$.[1]
Cohen Forcing / Easton Forcing:
$\text{Con(ZFC)} \Rightarrow \text{Con(ZFC} + \neg\text{GCH)}$.[2]

[1] Gödel (1940), Chapter VIII
[2] Cohen (1963 & 1964), Theorem 1(3), p.1143, and Easton (1970), Theorem 1, pp.140-1

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Example 2: SH

Suslin's Hypothesis (SH)

There is no Suslin line, that is, there is no linear order without endpoints that is dense and complete, in which there are at most countably many disjoint intervals, and which is not isomorphic to the real line.[3]

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Jensen in L: $\mathrm{Con(ZFC)} \Rightarrow \mathrm{Con(ZFC} + \neg \mathrm{SH})$.[4]
Solovay and Tennenbaum, iterated forcing:
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[3] Suslin in Sierpinski et al. (1920), p.223
[4] Jensen (1972), Theorem 6.2 and Lemma 6.5, pp.292–5
[5] Solovay and Tennenbaum (1971), Theorem 7.11, p.242

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Philosophical Questions

Independent Sentences in the Mathematical Account

According to the mathematical account of truth an independent sentence is neither true nor false.

Sentences that are neither true nor false?

Is it possible that there are mathematical statements that are neither true nor false?

New axioms?

- Should ZFC be extended by new axioms to reduce the independence phenomenon?
  - Which new axioms?
  - Why those?

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Method from Social Science

- Interview Study:
  1. Prepare and conduct the interviews
  2. Transcribe the interviews
  3. Analyse the transcriptions
- Qualitative: Analysis is mainly based on interpretation.
- Open: Results of the study are hypotheses and their evidence.
- Interviews are anonymised.

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Specific Setting

Main Research Question

What do set theorists think about independence?

- 25–35 interview partners (so far: 22): professional set theorists
- Interviews take 20–65 min (current average: 36 min)

Exemplary Interview Questions

- Can you delineate your research area in contrast to other set-theoretic research areas?
- Some set theorists are looking for further axioms to extend ZFC; others think that such aims are pointless. Do you have an opinion on that?

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- In Social Science, one can only approach empirical questions.
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Three Languages

Each discipline is practised in its own language

- Philosophy is practised in a philosophy-language
- Mathematics is practised in a mathematics-language
- Social Science is practised in a Social Science-language

Transfers and Relations

- Philosophical questions are transferred to the Social Science-language.
- Interview questions are formulated in the mathematics-language.
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- Most questions and results contain mathematical notions.

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- Results from Social Science are transferred back into the philosophy-language.
- Most questions and results contain mathematical notions.

Three Languages

Intermediate concept

Intermediate concept: Set-theoretic independence

- 'Set-theoretic independence' can in all three disciplines be explicated by the same mathematical and logical facts.
    - In philosophy, it is an attractive phenomenon to study.
    - In mathematics, it is a matter of fact.
    - In Social Science, it is a topic that underlies many set-theoretic practices.

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Ambiguous concept: Truth

- The meaning of 'truth' depends on the discipline.
  - In philosophy, there are many different accounts of truth in mathematics (platonism, semantic realism, pluralism, nominalism etc.).
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Interdisciplinary Procedure

1. Philosophical Question: Is it possible that there are mathematical statements that are neither true nor false?
2. Assumption: Methods from Social Science can inform our philosophical work
3. Choose, learn and apply the method
4. Result of the study: Empirical account of truth in mathematics (among others)
5. Approach the philosophical question and use the additional empirical account of truth

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3. Choose, learn and apply the method [Social Science]
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5. Approach the philosophical question and use the additional empirical account of truth [Philosophy]

Final Step: Integrate Empirical Findings in Philosophy

Different accounts of truth in mathematics

Accounts of truth in mathematics

- One mathematical account of truth in mathematics
- One empirical account of truth in mathematics
- Different philosophical accounts of truth in mathematics

Relating these accounts

- The empirical account includes the mathematical account (Mathematicians believe what has been proven).
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Limits of the empirical research

What empirical research can do for us

The empirical work can inform the philosophy in that we are able to determine which account corresponds best to the practices.[6]

What empirical research cannot do for us

The empirical work cannot help us in general to single out the right account of truth in mathematics.[7]

[6] Note that the question which account corresponds best to the practices is clearly of philosophical nature and extends the methods of Social Science. Thus, there is a distinction between philosophy using empirical research and sociology of mathematics.

[7] Though, we could take a practice-favourable stance and take the right account to be the one which corresponds best to the practices. But that would have to be argued for with philosophical arguments.

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