# How does a qualitative interview study inform the philosophy of set theory?

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In the philosophy of mathematics of today, scholars show more and more interest in the practices of mathematicians. Philosophers want to understand how the mathematicians' day-to-day work looks like, for they think that the mathematical practices are relevant to their ideas on mathematics (see, for instance, the engagement of the Association for the Philosophy of Mathematical Practice[1]).

Following this attitude, a qualitative *interview study* (25 to 35 interview partners) was set up to investigate how set-theoretic practices look like—in particular the mathematical work on *set-theoretic independence*. If one does philosophy by incorporating results from empirical work, one has to explain how these results relate to philosophy. In our case, we are situated in a specific framework and tackle the following question:

*How does a qualitative interview study with professional set theorists inform the philosophy of set theory?*

We summarise in the present abstract how one can systematise the interplay of the different disciplines in this specific context. We, first, distinguish the kind of questions and the languages. And, second, we retrace the path from the philosophical question to the interview study, and back from the results of the study to their integration in the philosophy of set theory.

Philosophers mostly ask *non-empirical* questions. But in Social Science, we can only approach *empirical* questions. Hence, we have to relate non-empirical questions to empirical ones. Moreover, not only do the kind of questions between the disciplines differ significantly, but also the *languages* in which the disciplines are practised are distinct. We need to transfer philosophical questions into the language of Social Science, and results from Social Science into philosophical language. With regard to the languages, we even have to deal with a third discipline, mathematics, since we work in the philosophy of mathematics.

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[1]Cp. their website www.philmathpractice.org.

In our specific framework, there is, fortunately, an *intermediate concept*: The concept of *set-theoretic independence*. This mathematical phenomenon is part of all three languages: In mathematics, as a matter of fact; in philosophy as an attractive phenomenon to study since it raises deep questions about mathematical truth; and in Social Science as a topic that underlies many set-theoretic practices. In all cases, the concept of set-theoretic independence can be explicated by the same mathematical theorems; the meaning of 'set-theoretic independence' does not depend on the discipline.

In contrast hereto, the meaning of '*(mathematical) truth*' depends on the discipline. In mathematics, a standard view is that a statement is true if it was proven in the theory ZFC.[2] In philosophy, we have many different accounts of mathematical truth at our disposal (respectively given by platonism, semantic realism, nominalism, naturalism, and so on). And in Social Science, the concept of truth is not explicated; rather its use (by mathematicians) can be analysed.

Let us now start with a *philosophical question* that is raised by the phenomenon of set-theoretic independence. As a matter of fact, there are many set-theoretic statements that are neither provable nor refutable in ZFC. According to the mathematical standard view of truth (mentioned above), such *independent* statements can neither shown to be true, nor shown to be false. The following question is raised: *Is it possible that there are mathematical statements that are neither true nor false?* Obviously, an answer to that question depends on the explication of the concept of truth.

We next turn to set-theoretic practices and assume that *suitable methods of Social Science* can inform our philosophical work on set-theoretic independence. We analyse in a qualitative interview study with professional set theorists how they use the notion of truth and what they think about set-theoretic independence.[3]

Once, the study is finished, it provides *determinate hypotheses* about the use of the notion of truth by set theorists and about their thoughts on independence. Furthermore, we have empirical evidence for these hypotheses. This suggests an empirical account of the concept of truth in mathematics.

We come back to philosophy and take up our question on mathematical statements which may be neither true nor false. From our empirical work, we gained two insights: First, we have an additional suggestion for an account of truth. Second, we know what set theorists think about independent statements, that is, we know what set theorists would answer to our question. We can now integrate these findings in our philosophical work, for example in the following way:

We focus, here, on the concept of truth to illustrate how the results of the interview study can inform the philosophical work. There are, at that point, three different kinds of an account of mathematical truth available:

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[2] Please note that this only gives a sufficient condition, but no definition of truth.

[3] The actual study incorporates already 21 interview partners. The analysis will be completed before the end of this year.

* one mathematical account of truth (If ZFC $\vdash \varphi$, then $\varphi$ is true.)

* one empirical account of mathematical truth

* several philosophical accounts of mathematical truth.

We can first assume that the mathematical account is also part of the philosophical accounts. We can assume as well that the mathematical account is part of the empirical account because we assume that mathematicians use the notion of truth in a way that can extend but not contradict the mathematical account of truth. If we take the empirical account to be relevant for the philosophy, we can work out a philosophical account of truth that is built around the empirical account.

This brings us to the general issue on the possibilities and limits of empirical work for philosophy. Given the different philosophical accounts, the empirical work can inform the philosophy in that we are able to determine which account corresponds best to the practices.[4] However, the empirical work cannot help us in general to single out the right account.[5]

In a 30 minutes talk, the systematisation described in the present abstract will be presented in an extended form.

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[4]Here, we found a question that makes a philosophy of mathematics which takes sociological work to be relevant different from the sociology of mathematics, for the question which account corresponds best to the practices is clearly of philosophical nature, and extends the methods of Social Science.

[5]Though, we could take a practice-favorable stance and take the best account to be the one which corresponds best to practice. But that would have to be argued for with philosophical arguments.